Correction du devoir n°7 – TS

Ex 1 : 1) $a_1 = \frac{3+2i}{4-5i} \times \frac{4+5i}{4+5i} = \frac{2+23i}{41} = \frac{2}{41} + \frac{23}{41} i$

$a_2 = -3e^{i3\pi/2} + 3e^{i5\pi} = -3\left(\cos\left(\frac{3\pi}{2}\right) + i\sin\left(\frac{3\pi}{2}\right)\right) + 3\left(\cos 5\pi + i\sin 5\pi\right)$

$= -3 \times (0-1) + 3 \times (-1+0) = 3i - 3 = -3+3i$

2) $b_1 = 3\sqrt{3} + 3i \quad |b_1|^2 = 27 + 9 = 36 \iff |b_1| = 6$

$b_1 = 6\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right) = 6e^{i\pi/6}$

$b_2 = -2\left(\cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)\right) = 2\left(-\cos\left(\frac{2\pi}{3}\right) - i\sin\left(\frac{2\pi}{3}\right)\right)$

$= 2\left(\cos\left(\frac{-\pi}{3}\right) + i\sin\left(\frac{-\pi}{3}\right)\right) = 2e^{-i\pi/3}$

Ex 2 : $(E_1) : 2iz - \bar{z} = 2$

$z = x+iy$

$\bar{z} = x - iy$

$(E_1) \iff 2i(x+iy) - (x-iy) = 2$

$\iff (-x-2y) + i(2x+y) = 2$

$\iff \begin{cases} -x-2y = 2 \\ 2x+y = 0 \end{cases} \iff \begin{cases} 3x = 2 \\ y = -2x \end{cases}$

$\iff x = 2/3$ et $y = -4/3$

$\boxed{S = \left\{ \frac{2}{3} - \frac{4}{3}i \right\}}$

$(E_2) : 5z^2 + 2z = -1 \iff 5z^2 + 2z + 1 = 0 \quad \Delta = -16$

$\sqrt{-\Delta} = 4$

deux solutions

$\begin{cases} z_1 = \frac{-2+4i}{10} = -\frac{1}{5} + \frac{2}{5}i \\ z_2 = \frac{-1}{5} - \frac{2}{5}i \end{cases}$

$\boxed{S = \{z_1 ; z_2\}}$

Ex3 : $\mathcal{E}_1 = \{M(z) / \arg(z - 2i + 1) = \frac{2\pi}{3} (2\pi)\}$

soit $A(2i-1)$ $M(z) \in \mathcal{E}_1 \iff (\vec{u}, \overrightarrow{AM}) = \frac{2\pi}{3} (2\pi)$

$\mathcal{E}_1$ est une demi-droite d'origine A (exclu)

$\mathcal{E}_2 = \{M(z) / \arg\left(\frac{z-2i+1}{z-3+i}\right) = \pi (2\pi)\}$

soit $B(3-i)$ $M(z) \in \mathcal{E}_2 \iff (\overrightarrow{BM}, \overrightarrow{AM}) = \pi (2\pi)$

$\mathcal{E}_2$ est le segment ouvert $]AB[$

$\mathcal{E}_3$ $\{M(z) / |z+2+i| = |z-2i|\}$

soient $C(-2-i)$ et $D(2i)$ $M(z) \in \mathcal{E}_3 \iff CM = DM$

$\mathcal{E}_3$ est la médiatrice de $[CD]$

Ex4 : $z' = \frac{z+3}{z-i}$ $(z \neq i)$ $\mathcal{E} = \{M(z) / z'$ imaginaire pur$\}$

1) a) $z = x + iy$ $z' = x' + iy'$

$$z' = \frac{x+iy+3}{x+iy-i} = \frac{(x+3)+iy}{x+(y-1)i} \times \frac{x-(y-1)i}{x-(y-1)i}$$

$$= \frac{x(x+3) + y(y-1) + i[-(x+3)(y-1) + xy]}{x^2 + (y-1)^2}$$

donc $\boxed{x' = \frac{x^2+3x+y^2-y}{x^2+(y-1)^2}}$ et $\boxed{y' = \frac{x-3y+3}{x^2+(y-1)^2}}$

b) z' imaginaire pur $\iff x' = 0 \iff x^2 + 3x + y^2 - y = 0$

$\iff \left(x+\frac{3}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 = \frac{10}{4} \iff IM^2 = \frac{10}{4} \iff IM = \frac{\sqrt{10}}{2}$

I d'affixe $z_I = -\frac{3}{2} + \frac{1}{2}i$ $\mathcal{E}$ est le cercle de centre I de rayon $\frac{\sqrt{10}}{2}$ privé du point $B(i)$

c) $A(-3)$
$B(i)$

$M(z) \in \mathcal{E} \iff \arg\left(\frac{z+3}{z-i}\right) = \frac{\pi}{2} (\pi)$ et $z \neq i$

$\iff (\overrightarrow{BM}, \overrightarrow{AM}) = \frac{\pi}{2} (\pi)$ et $M \neq B$

$\mathcal{E}$ est le cercle de diamètre $[AB]$ privé de B